investigated in Maeda et al. (1999) Neuron 24:989-1002 play a role of exogenous buffers. The binding ratio dependent on $\left[\mathrm{Ca}^{2+}\right]_{i}$ was extensively more than 1000 (Fierro and Llano (1996) J Physiol 496:617-625). In addition, $\mathrm{Ca}^{2+}$ indicators cerebellar Purkinje cells contain $\mathrm{Ca}^{2+}$ buffers at high concentrations, and the binding ratio is In cells, $99 \%$ of the $\mathrm{Ca}^{2+}$ ions are buffered with endogenous $\mathrm{Ca}^{2+}$-binding proteins. In particular
 We assumed that molecules do not diffuse through the spine neck.
 We modeled $\mathrm{Ca}^{2+}$ release through the $\mathrm{IP}_{3} \mathrm{R} \mathrm{Ca}{ }^{2+}$ channels, $\mathrm{Ca}^{2+}$ leaks through the plasma

and a stimulatory site to be exposed. binding serves to switch the $\mathrm{Ca}^{2+}$ sensitivity of $\mathrm{IP}_{3} R$ s by causing inhibitory sites to be masked (1999) Curr Biol 9:1115-1118. $\mathrm{IP}_{3} \mathrm{R}$ opening requires sequential binding of $\mathrm{IP}_{3}$ and $\mathrm{Ca}^{2+}$. $\mathrm{IP}_{3}$ We developed a novel kinetic model of $\mathrm{IP}_{3} R$ s based on a conceptual model in Adkins and Taylor

Schell (2001) Nat Rev Mol Cell Biol 2:327-338 Calcium 22:321-331). For a review for the metabolism of inositol phosphates, see Irvine and $\mathrm{IP}_{3}$ degradation is extensively reviewed in a simulation study (Dunplot and Erneux (1997) Cell C, $\mathrm{IP}_{3}$ degradation by $\mathrm{IP}_{3}$ 3-kinase (IP3K) and $\mathrm{IP}_{3}$ 5-phosphatase (IP5P)
 Gq $\alpha$ binds to PLC $\beta$ to enhance the PLC $\beta$ efficiency of $\mathrm{IP}_{3}$ production. PLC $\beta$ has GTPase
 paper of measuring G-protein-coupled-receptor kinetics entirely, and they reported very small a $\beta \gamma$ complex ( $G \beta \gamma$ ). As far as we know, Fay et al. (1991) Biochemistry 30:5066-5075 is an only promote the exchange of GDP to GTP of Gq. An activated Gq splits into an $\alpha$ subunit (Gq $\alpha$ ) and Glu released at the synaptic cleft binds to mGluRs. Activated mGluRs in turn binds to $G q$ to A, Glutamate binds to mGluRs, resulting in Gq activation.


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Supplemental Figure 1C
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Supplemental Figure 1F

Supplemental Tables: Parameter Tables
Supplemental Table 1: Molecular numbers and concentrations in the initial condition

| ID | Group | Molecular Name | \# | Total \# | Conc. <br> ( $\mu \mathrm{M}$ ) | Total <br> Conc. | Notes |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A1 | mGluR | Glu | 0 | 300 | 0 | 250 | Glutamate released at the synaptic cleft should be removed within milliseconds because individual EPSC responses can be seen with high-frequency stimuli ( 100 Hz ), for instance, in Takechi et al. (1998) Nature 396:757-760. Thus, we estimated that the decaying time constant of Glu is 5 msec . The initial number of Glu that access mGluRs localized at the edge of PSD was set to 300 , because concentration in neurotransmitter at the edge of spine is less than that at the center of spine (Franks et al. (2003) J Neurosci $23: 3186-3195$ ). This amount of Glu is sufficient to activate most of the mGluRs. \# in the Supplemental Tables indicates the initial number of the molecules in its present state, and the total \# indicates the total number of the molecules in any state. |
| A2 | mGluR | mGluR | 10 | 18 | 8.3333 | 15 | Metabotropic glutamate receptor type 1. Although the characteristic of this receptor was examined in two first cloning papers (Masu et al. (1991) Nature 349:760-765; Houamed et al. (1991) Science $252: 1318-1321$ ), we cannot estimate [mGluR] because the receptors were overexpressed by functional expression in Xenopus oocytes. In Bhalla and Iyengar (1999) Scinece 283:381-387, they estimated $0.3 \mu \mathrm{M}$ mGluR in a cell, and we took this value. Since the volume of the cytosol in the spine is $0.1 \mu \mathrm{~m}^{3}$ in our model, the number of mGluRs was estimated to be 18. The number is similar to those of AMPARs and VGCCs in a hippocampal dendritic spine (Matsuzaki et al. (2001) Nat Neurosci 4:1086-1092; Sabatini and Svoboda (2000) Nature 408:589-593). We modeled that mGluRs localize at the PSD, which has $1 / 50$-fold |


| ID | Group | Molecular Name | \# | Total \# | Conc. <br> ( $\mu \mathrm{M}$ ) | Total <br> Conc. | Notes |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | volume of the cytosol. We obtained $8.3333 \mu \mathrm{M} \mathrm{mGluR}$ in the PSD. |
| A3 | mGluR | mGluR-Glu | 0 | 18 | 0 | 15 | mGluRs activated by Glu binding. |
| A4 | mGluR | Gq-GDP | 52 | 60 | 43.333 | 50 | Trimeric G-protein Gq family. In Bhalla and Iyengar (1999) Science 283:381-387, they estimated $1.0 \mu \mathrm{M}$ Gq in a cell. Since the cytosolic volume in the spine is $0.1 \mu \mathrm{~m}^{3}$, the number of Gq was estimated to be 60 . We obtained $50 \mu \mathrm{M}$ Gq in the PSD because the cytosol has 50 -fold volume of the PSD. |
| A5 | mGluR | mGluR-Gq | 8 | 18 | 6.6667 | 15 | mGluRs binding to Gq without Glu. It is not clear whether mGluRs and Gq form complex before ligand stimulation (small $K_{\mathrm{d}}$ ), or ligand stimulation on mGluRs leads to the binding of mGluRs and Gq (large $K_{\mathrm{d}}$ ). In this model, we assumed half of the mGluRs bind Gq before glutamate release. |
| A6 | mGluR | Glu-mGluR-Gq | 0 | 18 | 0 | 15 | Intermediated state for Gq activation of Glu-mGluR-Gq complex. |
| A7 | mGluR | Ga-GTP | 0 | 60 | 0 | 50 | Activated Gq $\alpha$ subunit. Gq-GTP binds PLC $\beta$ to enhance $\mathrm{IP}_{3}$ productivity of PLC $\beta$. |
| A8 | mGluR | Gbc | 0 | 60 | 0 | 50 | G-protein $\beta \gamma$ complex. In this simulation, there are no proteins activated by G $\beta \gamma$. |
| A9 | mGluR | Ga-GDP | 0 | 60 | 0 | 50 | Inactivated Gq $\alpha$ subunit. G $\alpha$-GDP rapidly binds $\mathrm{G} \beta \gamma$ to form a trimer. |
| B1 | PLC | PIP2 | 5000 | 5000 | 4166.7 | 4166.7 | Phosphatidylinositol-4,5-bisphosphate. Molecular biology of the cell $4^{\text {th }}$ edition says that 5000000 lipid molecules exist in a $1 \mu \mathrm{~m}^{2}$ area of the plasma membrane. Since $\mathrm{PIP}_{2}$ is a minor lipid (less than $1 \%$ ), the number of $\mathrm{PIP}_{2}$ in the PSD was estimated to be 5000 . |
| B2 | PLC | PLC-PIP2 | 42 | 50 | 35 | 41.667 | PLC $\beta$ subtype 4 . We modeled PLC $\beta$ to bind $\mathrm{PIP}_{2}$ before PLC activation by $\mathrm{Ca}^{2+}$, because $\mathrm{PIP}_{2}$ concentration is high enough to bind almost all PLC in saturation. Bhalla and Iyengar (1999) |


| ID | Group | Molecular Name | \# | Total \# | Conc. <br> ( $\mu \mathrm{M}$ ) | Total <br> Conc. | Notes |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | Science 283:381-387 estimated $0.8 \mu \mathrm{M}$ PLC $\beta$ in a cell. Since the cytosolic volume in the spine is $0.1 \mu \mathrm{~m}^{3}$, the number of Gq was estimated to be 50 . We obtained $42 \mu \mathrm{M}$ PLC $\beta$ in the PSD because the cytosol has 50 -fold volume of the PSD. |
| B3 | PLC | PLC-PIP2-Ca | 7.5 | 50 | 6.25 | 41.667 | Without Gq, PLC $\beta$ activity is very low. |
| B4 | PLC | PLC-PIP2-Gq | 0 | 50 | 0 | 41.667 | This state has no enzyme activity. PLC $\beta 4$ requires $\mathrm{Ca}^{2+}$ for activation. |
| B5 | PLC | PLC-PIP2-Ca-Gq | 0 | 50 | 0 | 41.667 | Fully activated form of PLC $\beta 4$. PLC $\beta$ hydrolyzes $\mathrm{PIP}_{2}$ into DAG and $\mathrm{IP}_{3}$. PLC $\beta 4$ activation is dependent on Gq, whereas some other PLC $\beta$ subtypes are not. |
| B6 | PLC | PLC-Ca | 0.5 | 50 | 0 | 41.667 | The intermediate states of PLC $\beta$ that do not bind $\mathrm{PIP}_{2}$. We |
| B7 | PLC | PLC-Ca-Gq | 0 | 50 | 0 | 41.667 | assumed that PLC $\beta$ in the basal states bind $\mathrm{PIP}_{2}$. |
| B8 | PLC | DAG | 0 | 0 | 0 | 0 | Diacylglycerol activates no enzyme in the model because we do not implement DAG-dependent enzymes such as protein kinase C. |
| B9 | PLC | IP3_PSD | 0.12 | 12 | 0.1 | 10 | $\mathrm{IP}_{3}$ is produced by PLC $\beta$ in the PSD and diffuses to the cytosol. |
| C1 | IP3deg | IP3_spine | 6 | 600 | 0.1 | 10 | We set the basal $\left[\mathrm{IP}_{3}\right]$ to $0.1 \mu \mathrm{M}$ in this simulation. [ $\mathrm{IP}_{3}$ ] measurement in living cells is difficult. The only report is Luzzi et al. (1998) J Biol Chem 273:28657-28662. They estimated $0.04 \mu \mathrm{M}$ $\mathrm{IP}_{3}$ in Xenopus Oocytes by using capillary electrophoresis. |
| C2 | IP3deg | IP3_3-kinase | 52 | 54 | 0.86667 | 0.9 | $\mathrm{IP}_{3} 3$-kinase, which phosphorylates $\mathrm{IP}_{3}$ to $\mathrm{IP}_{4}$. In Takazawa et al. (1989) Biochem J 261:483-488, 0.020 mg of protein was purified from 700 g of bovine brain tissue. The yield was $4.4 \%$ and the molecular weight was 35000 . Thus, $0.020 \mathrm{mg} \times(100 \% / 4.4 \%) /$ $(35000 \mathrm{~g} / \mathrm{mol}) / 0.7$ liter $=0.019 \mu \mathrm{M}$ while assuming the specific gravity of the tissue $1 \mathrm{~kg} / \mathrm{liter}$. This enzyme is highly localized in Purkinje dendritic spines (Yamada et al. (1993) Brain Res |


| ID | Group | Molecular Name | \# | Total \# | Conc. <br> ( $\mu \mathrm{M}$ ) | Total <br> Conc. | Notes |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | 606:335-340; Go et al. (1993) Neurosci Lett 158:135-138). Therefore, we increased [IP3K] to $0.9 \mu \mathrm{M}$. |
| C3 | IP3deg | IP3K-2Ca | 2 | 54 | 0.033333 | 0.9 | $\mathrm{Ca}^{2+-}$ bound state of IP3K. |
| C4 | IP3deg | IP3K-2Ca-IP ${ }_{3}$ | 0 | 54 | 0 | 0.9 | $\mathrm{Ca}^{2+-}$ and $\mathrm{IP}_{3}$-bound state of IP3K. |
| C5 | IP3deg | IP3_5-phos | 58.8 | 60 | 0.98 | 1 | $\mathrm{IP}_{3}$ 5-phosphatase, which dephosphorylates $\mathrm{IP}_{3}$ to $\mathrm{IP}_{2}$. From Verjans et al. (1992) Eur J Biochem 204:1083-1087, 0.806 mg of IP5P was obtained from 2 kg of brain tissue. The yield was $15 \%$ and the molecular weight was 43,000 . Thus, 0.806 mg x $(100 \% / 15 \%) /(43000 \mathrm{~g} / \mathrm{mol}) / 2$ liter brain $=0.06 \mu \mathrm{M}$, while assuming the specific gravity of the tissue $1 \mathrm{~kg} /$ liter. A study using antibodies showed that the enzyme was highly expressed in the Purkinje neurons (De Smedt et al. (1996) JBC 271:10419-10424). Thus, we increased it to $1 \mu \mathrm{M}$. |
| C6 | IP3deg | IP5P-IP3 | 1.2 | 60 | 0.02 | 1 | Intermediate binding state of $\mathrm{IP}_{3} 5$-phosphatase and $\mathrm{IP}_{3}$. |
| D1 | IP3R | IP3Rec | 14.22 | 16 | 0.237 | 0.26667 | $\mathrm{IP}_{3} \mathrm{R}$ type 1 is highly expressed in Purkinje cells. We counted 16 immunogold spots in the figure of the PF spine slice in Otsu et al. (1990) Cell Struct Function 15:163-173. Since the slice has $1 / 4$ thickness of a spine and $\mathrm{IP}_{3} \mathrm{Rs}$ are homotetramers, the number of $\mathrm{IP}_{3} \mathrm{Rs}$ in a PF spine was estimated to be $16(=16 /(1 / 4) / 4)$. |
| D2 | IP3R | IP3R-IP3 | 0.06 | 16 | 0.1 | 0.26667 | $\mathrm{IP}_{3}$-bound state of $\mathrm{IP}_{3} \mathrm{Rs}$. In our $\mathrm{IP}_{3} \mathrm{R}$ kinetics model, $\mathrm{IP}_{3}$ binding to $\mathrm{IP}_{3}$ Rs allows activation of the $\mathrm{IP}_{3} \mathrm{Rs}_{\text {s }}$ by $\mathrm{Ca}^{2+}$. |
| D3 | IP3R | IP3R_open | 0.01 | 16 | 0.00016667 | 0.26667 | Open state of $\mathrm{IP}_{3} \mathrm{Rs}$. |
| D4 | IP3R | IP3R-Ca | 1.5 | 16 | 0.025 | 0.26667 | Inactivation state of $\mathrm{IP}_{3} \mathrm{Rs}$, bound to one $\mathrm{Ca}^{2+}$ ion. |
| D5 | IP3R | IP3R-2Ca | 0.18 | 16 | 0.003 | 0.26667 | Inactivation state of $\mathrm{IP}_{3} \mathrm{Rs}$, bound to two $\mathrm{Ca}^{2+}$ ions. |
| D6 | IP3R | IP3R-3Ca | 0.03 | 16 | 0.0005 | 0.26667 | Inactivation state of $\mathrm{IP}_{3} \mathrm{Rs}$, bound to three $\mathrm{Ca}^{2+}$ ions. |
| D7 | IP3R | IP3R-4Ca | 0 | 16 | 0 | 0.26667 | Inactivation state of $\mathrm{IP}_{3} \mathrm{Rs}$, bound to four $\mathrm{Ca}^{2+}$ ions. |


|  | ID | Group | Molecular Name | \# | Total \# | Conc. <br> ( $\mu \mathrm{M}$ ) | Total <br> Conc. | Notes |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | E1 | CaReg | CaSpine | 3.6 | 3.6 | 0.06 | 0.06 | Free cytosolic $\mathrm{Ca}^{2+}$ concentration, $\left[\mathrm{Ca}^{2+}\right]_{\mathrm{i}}$. The basal $\left[\mathrm{Ca}^{2+}\right]_{\mathrm{i}}$ is set to $0.06 \mu \mathrm{M}$. |
|  | E2 | CaReg | Ca2+PSD | 0.072 | 0.072 | 0.06 | 0.06 | $\mathrm{Ca}^{2+}$ concentration in the postsynaptic density. This concentration is used only for $\mathrm{Ca}^{2+-}$ dependent $\mathrm{PLC} \beta$ activation. It has a slight effect on $\mathrm{IP}_{3}$ productivity of $\mathrm{PLC} \beta$ in the PSD. |
|  | E3 | CaReg | SERCA | 148 | 155 | 2.4667 | 2.5833 | Sacro ${ }^{-}$and endoplasmic reticulum $\mathrm{Ca}^{2+}$ ATPase. SERCA is a dominant protein in the ER, constituting $80 \%$ of the ER membrane protein (Stryer Biochemistry $5^{\text {th }}$ edition). SERCA type 2 is dominant in the Purkinje cells (Takei et al. (1992) J Neurosci $12: 489-505)$. We assumed the number of SERCA so that $\left[\mathrm{Ca}^{2+}\right]_{\mathrm{ER}}$ is $150 \mu \mathrm{M}$ at the basal state. |
|  | E4 | CaReg | SERCA-2Ca | 7 | 155 | 0.11667 | 2.5833 | $\mathrm{Ca}^{2+-}$ bound state of SERCA. |
|  | E5 | CaReg | PMCA | 68 | 108 | 1.1333 | 1.8 | Plasma membrane $\mathrm{Ca}^{2+}$-ATPase. Type 2 of PMCA is abundant in Purkinje cells (de Talamoni et al. (1993) PNAS 90:11949-11953). PMCA has higher affinity and lower capacity than $\mathrm{Na}^{+} / \mathrm{Ca}^{2+}$ exchangers. At the basal $\left[\mathrm{Ca}^{2+}\right]_{\mathrm{i}}$, PMCA pumps out much more $\mathrm{Ca}^{2+}$ ions than $\mathrm{Na}^{+} / \mathrm{Ca}^{2+}$. We chose $[\mathrm{PMCA}]$ so that $\left[\mathrm{Ca}^{2+}\right]_{\mathrm{i}}$ is $0.06 \mu \mathrm{M}$ at the basal state. |
|  | E6 | CaReg | PMCA-Ca | 40 | 108 | 0.66667 | 1.8 | $\mathrm{Ca}^{2+}$-bound state of PMCA. |
|  | E7 | CaReg | NCX | 32 | 32 | 0.53333 | 0.53333 | $\mathrm{Na}^{+} / \mathrm{Ca}^{2+}$ exchangers. They use $\mathrm{Na}^{+}$electrochemical gradient across the plasma membrane as their energy source. Note that we do not model membrane potential. NCXs play a major role in pumping out intracellular $\mathrm{Ca}^{2+}$ at micromolar levels of $\left[\mathrm{Ca}^{2+}\right]_{\mathrm{i}}$,. |
|  | E8 | CaReg | NCX-2Ca | 0 | 32 | 0 | 0.53333 | $\mathrm{Ca}^{2+}$-bound state of $\mathrm{Na}^{+} / \mathrm{Ca}^{2+}$ exchangers. |
| $\stackrel{\rightharpoonup}{N}$ | E9 | CaReg | CaStore | 1800 | 30000 | 150 | 2500 | Free $\mathrm{Ca}^{2+}$ concentration in the $\mathrm{ER},\left[\mathrm{Ca}^{2+}\right]_{\mathrm{ER}}$, was previously assumed to be more than 1 mM in 1990s (for example, Fiala et al. |


| ID | Group | Molecular Name | \# | Total\# | Conc. <br> $(\mu \mathrm{M})$ | Total <br> Conc. | Notes |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |


| ID | Group | Molecular Name | \# | Total \# | Conc. <br> ( $\mu \mathrm{M}$ ) | Total <br> Conc. | Notes |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | Although Maeda et al. (Neuron 24:989-1002 (1999)) studied the effect of $\mathrm{Ca}^{2+}$ buffers in cerebellar Purkinje cells and estimated 360 $\mu \mathrm{M}$ [CB], they neglected effects of other high-affinity buffers and $\mathrm{Ca}^{2+}$ pumps dependent on $\left[\mathrm{Ca}^{2+}\right]_{\mathrm{i}}$. [CB] was estimated to be $100 \mu \mathrm{M}$ in our simulation. |
| F6 | CaBuf | CB-2Ca | 150 | 6000 | 2.5 | 100 | $\mathrm{Ca}^{2+-}$ bound state of calbindin- $\mathrm{D}_{28 \mathrm{k}}$. |
| F7 | CaBuf | LowAffBuf | 5997 | 6000 | 99.95 | 100 | Purkinje cells contain low-affinity buffers at high concentrations (Maeda et al. (1999) Neuron 24:989-1002). We fitted the binding ratio in our model to the binding ratio in their Fig. 6 by using two buffers. Low-affinity buffer 1 (LAB) is non-cooperative, and Hill coefficient is 1 . |
| F8 | CaBuf | LAB-Ca | 3 | 6000 | 0.05 | 100 | $\mathrm{Ca}^{2+-}$ bound state of low-affinity buffer 1. |
| F9 | CaBuf | LowAffBuf2 | 6000 | 6000 | 100 | 100 | Purkinje cells contain low-affinity buffers at high concentration (Maeda et al. (1999) Neuron 24:989-1002). We fitted the biding ratio curve in their Fig. 6 by using two buffers. Low-affinity buffer 2 (LAB2) is cooperative, and the Hill coefficient is 2. |
| F10 | CaBuf | LAB2-2Ca | 0 | 6000 | 0 | 100 | $\mathrm{Ca}^{2+-}$ bound state of low-affinity buffer 2 . |

Supplemental Table 2: Kinetic rate constants in the reactions



|  | ID | Group | Reaction Name | $k_{f}$ | $k_{b}(/ \mathrm{sec})$ | $K_{\text {d }}(\mu \mathrm{M})$ | Notes |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | Cell Calcium 22:321-331). $K_{\mathrm{m}}$ for $\mathrm{Ca}^{2+}=0.3 \mu \mathrm{M}$ and Hill coefficient $=2$. |
|  | c2 | IP3deg | IP3K-2Ca_bind_IP3 | 100 | 80 | 0.8 | We took the $K_{\mathrm{m}}$ value from a $\mathrm{Ca}^{2+}$ simulation paper (Dunplot and Erneux (1997) Cell Calcium 22:321-331). Michaelis constant $K_{\mathrm{m}}$ is $1 \mu \mathrm{M} . K_{\mathrm{m}}=\left(k_{b}+k_{c a t}\right) / k_{f}=$ $(80+20) / 100=1 \mu \mathrm{M}$. |
|  | c3 | IP3deg | IP3K_deg_IP3 | 20 |  |  | Several studies reported very different $V_{\max }$ values (Irvine et al. (1986) Nature 320:631-634; Takazawa et al. (1989) Biochem J 261:483-488; Choi et al. (1990) Science $248: 64-66$ ). Thus, we did not take any reported value from these studies. |
|  | c4 | IP3deg | IP5P_bind_IP3 | 9 | 72 | 8 | We took the $K_{\mathrm{m}}$ value from a $\mathrm{Ca}^{2+}$ simulation paper (Dunplot and Erneux (1997) Cell Calcium 22:321-331). Michaelis constant $K_{\mathrm{m}}=1 \mu \mathrm{M} . K_{\mathrm{m}}=\left(k_{b}+k_{c a t}\right) / k_{f}=$ $(72+18) / 9=10 \mu \mathrm{M}$. |
|  | c5 | IP3deg | IP5P_deg_IP3 | 18 |  |  | A purification study reported that $V_{\max }=20-35 \mu \mathrm{~mol} / \mathrm{min} / \mathrm{mg}$ protein (Verjans et al. (1992) Eur J Biochem 204:1083-1087). IP 3 5-phosphatase is a 43 kDa enzyme, and we obtained $k_{c a t}=20-35 \times 43000 / 60000=18 \# / s e c / \#$ protein. This unit conversion method was described in De Schutter (2000) Computational Neuroscience, CRC Press, Boca Raton, pp. 31. |
|  | d1 | IP3R | IP3R_bind_IP3 | 1000 | 25800 | 25.8 | From measurement of $\mathrm{Ca}^{2+}$ depletion of the ER stores, the affinity of $\mathrm{IP}_{3} \mathrm{Rs}$ for $\mathrm{IP}_{3}$ in Purkinje cells is much lower ( $K_{\mathrm{d}}=25.8 \mu \mathrm{M}$ ) than in vitro (Fujiwara et al. (2001) Neuroreport 12:2647-2651). The low affinity is consistent with the fact that $\mathrm{Ca}^{2+}$ response to $\mathrm{IP}_{3}$ uncaging in Purkinje cells require strong photostimulus ( $\left[\mathrm{IP}_{3}\right]>10 \mu \mathrm{M}$ ) for (Khodakhah and Ogden (1993) PNAS 90:4976-4980; Finch and Augustine (1998) Nature 396:753-756). Thus, we used $K_{\mathrm{d}}$ of $25.8 \mu \mathrm{M}$ in the simulation. |
| $\stackrel{\rightharpoonup}{*}$ | d2 | IP3R | IP3R-IP3_bind_Ca | 8000 | 2000 | 0.25 | $\mathrm{Ca}^{2+}$ directly binds to $\mathrm{IP}_{3} \mathrm{Rs}$ for activation. From a bell-shaped $\mathrm{Ca}^{2+}$-dependency of $\mathrm{IP}_{3}$ Rs (Bezprozvanny and Ehric (1994) J Gen Physiol 104:821-856; Fujiwara et al. (2001) Neuroreport 12:2647-2651), we obtained $K_{\mathrm{d}}=0.25 \mu \mathrm{M}$. The reaction must be faster than $\mathrm{Ca}^{2+}$-dependent $\mathrm{IP}_{3} \mathrm{R}$ inactivation for $\mathrm{Ca}^{2+}$ positive feedback. |


|  | ID | Group | Reaction Name | $k_{f}$ | $k_{b}(/ \mathrm{sec})$ | $K_{\text {d }}(\mu \mathrm{M})$ | Notes |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | d3 | IP3R | IP3R_bind_Ca | 8.889 | 5 | 0.56249 | $\mathrm{IP}_{3}$ Rs are completely inactivated at high concentrations of $\left[\mathrm{Ca}^{2+}\right]_{\mathrm{i}}(<10 \mu \mathrm{M})$. In our kinetic model, an $\mathrm{IP}_{3} \mathrm{R}$ has four $\mathrm{Ca}^{2+}$ inactivation sites. This sequential $\mathrm{Ca}^{2+}$ binding reaction was assumed to be positively cooperative. In other words, $\mathrm{Ca}^{2+}$ ions bind a subunit easier as more $\mathrm{Ca}^{2+}$ binds to $\mathrm{IP}_{3} \mathrm{Rs}$. How to estimate these parameters is written in $\mathrm{Ca}^{2+-}$ dependent $\mathrm{IP}_{3} \mathrm{R}$ inactivation in the Material and Methods. |
|  | d4 | IP3R | IP3R-Ca_bind_Ca | 20 | 10 | 0.5 |  |
|  | d5 | IP3R | IP3R-2Ca_bind_Ca | 40 | 15 | 0.375 |  |
|  | d6 | IP3R | IP3R-3Ca_bind_Ca | 60 | 20 | 0.33333 |  |
|  | e1 | CaReg | IP3R_Ca_channel | 450 | 450 | (perm) | In Bezprozvanny and Ehrich (1994) J Gen Physiol 104:821-856, they estimated that $5400 \mathrm{Ca}^{2+}$ ions go through an open $\mathrm{IP}_{3} \mathrm{R}$ per second at $2500 \mu \mathrm{M}\left[\mathrm{Ca}^{2+}\right]_{\text {ER }}$. The unit of permability is not described in GENESIS/kinetikit, but we found that the parameter value of 450 matches the estimation. |
|  | e2 | CaReg | SERCA_bind_2Ca | 17147 | 1000 | 0.24149 | We took the kinetics parameter of SERCA subtype 2b from Lytton et al. (1992) JBC 267:14483-14489. $K_{\mathrm{m}}=0.27 \mu \mathrm{M}$ and the Hill coefficient $=2 . K_{\mathrm{m}}{ }^{2}=\left(k_{b}+\right.$ $\left.k_{c a t}\right) / k_{f}=(1000+250) / 17147=(0.27 \mu \mathrm{M})^{2}$. |
|  | e3 | CaReg | SERCA_uptake | 250 |  |  | In Stryer Biochemistry $5^{\text {th }}$ edition, one SERCA pumps out less than $100 \mathrm{Ca}^{2+}$ ions per second. Thus, we took $k_{\text {cat }}$ value to be $50 / \mathrm{s}$ at first, but the low kcat value caused a problem. The speed of $\mathrm{Ca}^{2+}$ clearance does not increase in proportion to the number of $\mathrm{Ca}^{2+}$ pumps because free $\mathrm{Ca}^{2+}$ concentration becomes low and the $\mathrm{Ca}^{2+}$ binding to the pump decreases in the large presence of $\mathrm{Ca}^{2+}$ pumps with low $k_{c a t}$. We failed to reproduce $\mathrm{Ca}^{2+}$ time course showed in Wang et al. (2000) Nat Neurosci 3:1266-1273 even if we increased the number of pumps. Following the fact that most of the other $\mathrm{Ca}^{2+}$ simulation neglect the binding effect of $\mathrm{Ca}^{2+}$ pumps, we decided to reduce the buffing effect by adjusting $k_{c a t}$ and [SERCA], while keeping $V_{\max }\left(=k_{c a t} \mathrm{x}\right.$ [SERCA]). Thus, we increased $k_{\text {cat }} 5$-fold and decreased [SERCA] $1 / 5$-fold. |
|  | e4 | CaReg | Ca_Leak_from_ER | 15 | 15 | (perm) | This leak parameter was dependent on other parameters of $\mathrm{Ca}^{2+}$ regulation. |
| $\infty$ | e5 | CaReg | PMCA_bind_Ca | 25000 | 2000 | 0.08 | We took the kinetic parameters of PMCA subtype 2 from Stauffer et al. (1995) JBC 270:12184-12190. $K_{\mathrm{m}}=0.1 \mu \mathrm{M}$ and Hill coefficient $=1 . K_{\mathrm{m}}=\left(k_{b}+k_{c a t}\right) / k_{f}=$ $(500+2000) / 25000=0.1 \mu \mathrm{M}$. |


| ID | Group | Reaction Name | $k_{f}$ | $k_{b}(/ \mathrm{sec})$ | $K_{\text {d }}(\mu \mathrm{M})$ | Notes |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| e6 | CaReg | PMCA_uptake | 500 |  |  | The capacity of PMCA was increased to 10 times ( 50 to 500 ) for the same reason as the increase in $k_{c a t}$ for SERCA. |
| e7 | CaReg | NCX_bind_2Ca | 93.827 | 4000 | 6.5293 | Since this model does not include voltage, the efficacy of $\mathrm{Na}^{+} / \mathrm{Ca}^{2+}$ uptake should be dependent only on $\mathrm{Ca}^{2+}$ concentration, not on the membrane potential. In Fujioka et al. (2000) J Physiol 529:611-623, they measured $\mathrm{Ca}^{2+-}$ dependent current of $\mathrm{Na}^{+} / \mathrm{Ca}^{2+}$. $K_{\mathrm{m}}$ for $\mathrm{Ca}^{2+}=7.3 \mu \mathrm{M}$ and the Hill coefficient $=2 . K_{\mathrm{m}}{ }^{2}=\left(k_{b}+\right.$ $\left.k_{\text {cat }}\right) / k_{f}=(4000+1000) / 93.287=(7.3 \mu \mathrm{M})^{2}$. |
| e8 | CaReg | NCX_uptake | 1000 |  |  | Stryer Biochemistry $5^{\text {th }}$ edition says that a $\mathrm{Na}^{+} / \mathrm{Ca}^{2+}$ can extrude $2,000 \mathrm{Ca}^{2+}$ ions per second. Since the Hill coefficient is 2 , a $\mathrm{Na}^{+} / \mathrm{Ca}^{2+}$ transports $2 \mathrm{Ca}^{2+}$ ions at one reaction cycle. Thus, $k_{c a t}=2000 / 2=1000 / \mathrm{s}$. |
| e9 | CaReg | Ca_Leak_from_ext | 10 | 10 | (perm) | This leak parameter was dependent on other parameters of $\mathrm{Ca}^{2+}$ regulation. |
| e10 | CaReg | Ca_bind_calreticulin | 0.1 | 200 | 2000 | Calreticulin is a high-concentration and uncooperative buffer. We took $K_{\mathrm{d}}$ to be 2 mM , according to Krause and Michalak (1997) Cell 88:439-443. |
| f1 | CaBuf | Ca_bind_MgGreen | 1000 | 19000 | 19 | Apparent $K_{\mathrm{d}}$ of Magnesium Green 1 for $\mathrm{Ca}^{2+}$ is $19 \mu \mathrm{M}$ in the presence of endogenous $\mathrm{Mg}^{2+}$, from Wang et al. (2000) Nat Neurosci 3:1266-1273. |
| f2 | CaBuf | PV_bind_Ca | 18.5 | 0.95 | 0.05315 | Parvalbumin is a slow and high-affinity buffer. Most of the parvalbumin binds endogenous $\mathrm{Mg}^{2+}$ at the basal $\mathrm{Ca}^{2+}$ concentration,. In Lee et al. (2000) J Physiol 525:419-431, they measured the apparent dissociation constant $K_{d}=0.0514 \mu \mathrm{M}$ and the unbinding rate constant $k_{b}=0.95 / \mathrm{sec}$ in the presence of $\mathrm{Mg}^{2+}$, and we used the parameter values. |
| f3 | CaBuf | CB_bind_2Ca | 87 | 11.275 | 0.36 | Calbindin is a cooperative and high-affinity buffer. We took the $K_{d}$ value and the Hill coefficient to be $0.36 \mu \mathrm{M}$ and 2, from Maeda et al. (1999) Neuron 24:989-1002. These values were consistent with (2:2) ratio in Table 1 in Nagerl et al. (2000) Biophys J 79:3009-3018. |
| f4 | CaBuf | LAB_bind_Ca | 10 | 1000 | 100 | We included low-affinity buffers based on Fig. 6 in Maeda et al. (1999) Neuron 24:989-1002. The Hill coefficients of low-affinity buffer 1 (LAB) and low-affinity |
| f5 | CaBuf | LAB2_bind_2Ca | 1 | 4000 | 20 | buffer 2 (LAB2) were taken 1 and 2, respectively. |

Supplemental Table 3: Categories of parameters

|  | known |  |  |  | unknown |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Form Purkinje cells | From other cells | From reaction in test tubes | From other molecular subtypes |  |
| Time constants, $\tau$ |  | f2, f3 | $\begin{gathered} \text { a6, a8, } \\ \text { e1, e4 ,e9 } \end{gathered}$ |  | $\begin{gathered} \mathrm{a} 1, \mathrm{a} 2, \mathrm{a} 3, \mathrm{a} 4, \mathrm{~b} 1, \mathrm{~b} 2, \mathrm{~b} 3, \\ \mathrm{~b} 4, \mathrm{~b} 5, \mathrm{~b} 8, \mathrm{~b} 9, \mathrm{c} 1, \mathrm{c} 2, \mathrm{c} 4, \\ \mathrm{~d} 1, \mathrm{~d} 2, \mathrm{~d} 3, \mathrm{~d} 4, \mathrm{~d} 5, \mathrm{~d} 6, \\ \mathrm{e} 2, \mathrm{e} 5, \mathrm{e} 7, \mathrm{e} 10, \mathrm{f} 1, \mathrm{f} 4, \mathrm{f} 5 \end{gathered}$ |
| Dissociation constants, $K_{\mathrm{d}}$ and Michaelis constants, $K_{\mathrm{m}}$ | d1, d2, d3, d4, d5,d6 | $\begin{gathered} \hline \mathrm{a} 1, \mathrm{a} 2, \mathrm{a} 4, \\ \mathrm{f} 1, \mathrm{f} 2, \mathrm{f} 3 \end{gathered}$ | $\begin{gathered} \mathrm{b} 4, \mathrm{~b} 5, \mathrm{c} 1, \mathrm{c} 2, \mathrm{c} 4, \\ \mathrm{e} 2, \mathrm{e} 5, \mathrm{e} 7, \mathrm{e} 10 \end{gathered}$ | $\begin{gathered} \hline \mathrm{b} 1, \mathrm{~b} 2, \mathrm{~b} 3, \\ \mathrm{~b} 8, \mathrm{~b} 9 \end{gathered}$ | a3, f4 , f5 |
| Enzyme turnovers, $V_{\text {max }}$ |  | $\begin{gathered} \text { b10, b11, } \\ \text { b12 } \end{gathered}$ | a7, c5, e3, e6, e8 | b7 | a5, b6, c3 |
| Initial concentrations, [] | $\left[\mathrm{Ca}^{2+}\right]_{\mathrm{i}}, \mathrm{IP}_{3} \mathrm{R}$, MgGreen, parvalbumin, calbindin- $\mathrm{D}_{28 \mathrm{k}}$, low-affinity buffer, low-affinity buffer 2 | $\begin{gathered} \text { Glu, } \\ \text { mGluR, } \mathrm{Gq}, \\ \mathrm{IP}_{3}, \\ {\left[\mathrm{Ca}^{2+}\right]_{\mathrm{ER}},} \\ {\left[\mathrm{Ca}^{2+}\right]_{\text {ext }},} \\ \text { calreticulin } \end{gathered}$ | $\mathrm{PIP}_{2}$, <br> $\mathrm{IP}_{3} 3$-kinase, $\mathrm{IP}_{3} 5$-phosphatase | PLC $\beta 4$ <br> (taken from <br> PLC $\beta 1$ ) | SERCA, <br> PMCA, <br> $\mathrm{Na}^{+} / \mathrm{Ca}^{2+}$ |

$$
\mathrm{IP}_{3} \text { time course and time window are robust against variations of the parameter value. }
$$ unknown). When the symbols of one parameter gather together on the dashed line (control), the dark color: increase) and whether the parameter is known or unknown (blue: known; red

 ( $\bigcirc: 1 / 2$ or $2 ; \triangle: 1 / 5$ or $5 ; \nabla: 1 / 10$ or $10 ; \square: 1 / 20$ or $20 ; \diamond: 1 / 50$ or $50 ; \pm: 1 / 100$ or 100 ). The
 ( $($ ( ) suоппедииәэuoэ dissociation and Michaelis constants (I), maximum enzyme velocities (J), and initial molecular $H-K$, The three features of time window when we varied the values of time constants $(H)$, the time window. The time windows were analyzed in the time interval from -500 to 600 ms . three features: the peak time, half-width, and peak amplitude of the Gaussian function fitted to $1 / 20,1 / 50$, and $1 / 100$ times (pink and broken lines). G, For quantitative comparisons, we used Glu and mGluR was varied to $2,5,10,20,50$, and 100 (red and bold lines) and $1 / 2,1 / 5,1 / 10$ the $\mathrm{IP}_{3}$ time course. $F$, An example of re-computed time window. The time constant for binding the values of all other parameters, and the time window function was re-computed as well as known and unknown parameters was varied in a range of 1/100-100 times while maintaining

 time course when we varied the values of time constants $(B)$, dissociation and Michaelis $1 / 50$, and $1 / 100$ times (pink and broken lines). $B-E$, The peak time and peak amplitude of $I P_{3}$ Glu-mGluR was varied to $2,5,10,20,50$, and 100 (red and bold lines) and $1 / 2,1 / 5,1 / 10,1 / 20$, example of re-computed $\mathrm{IP}_{3}$ time course. The maximum enzyme velocity for Gq activation by all other parameters, and the $\mathrm{IP}_{3}$ time course with PF-alone input was re-computed. $A$, An






## function.

Analysis of parameter sensitivity to the $I P_{3}$ time course and time window

## Supplemental Figure 2


Peak amplitude of IP3 time course






|  | x1／100 x1／50 x1／20 | x1／10 x $1 / 5 \times 1 / 2$ | x2 | x5 | x10 | x20 | x50 | $\times 100$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| known | 分 $\quad \square$ | $\nabla \triangle \bigcirc$ | $\bigcirc$ | $\triangle$ | $\nabla$ | $\square$ | $\rangle$ | K |
| unknown | ふく | $\nabla \triangle \bigcirc$ | $\bigcirc$ | $\triangle$ | $\nabla$ | $\square$ | $\rangle$ | 令 |





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Supplemental Figure 2G
Averaged $\mathrm{Ca}^{2+}$ response (\% $\Delta F / F_{0}$ )


Supplemental Figure 2F
$\Delta \mathrm{F} / \mathrm{F} 0$ (\%)


Time (ms)


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$\Delta$ F/Fo (\%)
Time (ms)




Time (ms)


Time (ms)


Supplementa
Figure 2K

