Supplemental Information

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Bayesian Adaptive Psychophysical Method

Our implementation of the psi method of Kontsevich and Tyler (1999) is described in detail in Goldreich et al. (2009). Briefly, we modeled d-prime as a power function of groove width, x,

$$d' = \left(\frac{x}{a}\right)^b$$

and each participant's psychometric function as a mixture of a cumulative normal curve and a lapse rate term:

$$P_c(x) = \frac{\delta}{2} + (1-\delta)\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{d'/\sqrt{2}} \exp\left(-\frac{y^2}{2}\right)dy$$

We used the psi method to conduct the grating orientation task, presenting in each trial the groove width that maximized expected information gain. We modified the algorithm by treating not only *a* (threshold) and *b* (slope) but also δ (lapse rate) as unknown parameters. Using uniform prior probabilities over a range of *a* (0.5 – 3.0), *b* (0.01 – 10.0), and δ (0.01 – 0.10) values, we calculated the joint posterior probability density function (PDF) for the participant's psychometric function (*a, b,* δ) and marginalized this over *b* and δ to generate the posterior PDF for the *a*-parameter, the threshold stimulus level corresponding to 76% correct response probability (d-prime = 1).

Statistical Analysis

For conventional statistical analyses (t-test, correlation, ANCOVA, regression), we used as the dependent measure the mode of each participant's *a*-parameter posterior PDF, a point estimate of grating orientation threshold.

For Bayesian analyses, we computed Bayes factors (marginal likelihood ratios) for each of three alternative models compared to a null model. An advantage of the Bayesian approach is that it naturally embodies an *Occam factor* penalty against unprofitably complex models (Sivia and Skilling, 2006).

To enhance the power of the Bayesian analysis, models 1, 2, and 3 (see below) specified one-sided effects (e.g., women have lower thresholds than men; finger size correlates positively with threshold), all four models utilized not simply the point-estimate threshold for each participant but the participant's entire sequence of correct and incorrect responses at the groove widths tested, and all models incorporated robust outlier-resistance. The models, and methods for computing their likelihoods, are described below.

Null model (M_0)

According to the null model, all participants (regardless of sex or finger size) belong to the same Gaussian population distribution, characterized by a single mean tactile acuity and standard deviation. The expected 76% threshold (a-parameter) for each participant, then, is

$$\hat{a} = C_1$$

where C_1 is the (unknown) population mean.

To determine the probability of the data according to this model, we first find the probability of each participant's data (correct and incorrect responses at each groove width), d_i, according to the model with a particular C_1 and σ:

$$P(d_i \mid M_0, C_1, \sigma) = \sum_a P(d_i \mid a) \bullet P(a \mid M_0, C_1, \sigma) \quad (Eqn.1)$$

Note that, as mentioned above, we do not use the mode of a participant's *a*-parameter posterior PDF as the datum for that participant. Rather, recognizing that the participant's *a*-parameter is not known with certainty, we marginalize (integrate) over all possible values of the *a*-parameter. This procedure enhances the power of the analysis, effectively giving more inferential weight to participants with sharper *a*-parameter PDFs (i.e., participants who responded most consistently during the experiment).

The first term on the right side of equation (1) is

$$P(d_i \mid a) = \sum_{b,\delta} P(d_i \mid a,b,\delta) \bullet P(b,\delta \mid a) \propto \sum_{b,\delta} \prod_x \left[p_{cx}(a,b,\delta)\right]^{n_{rx}} \bullet \left[1 - p_{cx}(a,b,\delta)\right]^{n_{wx}} \quad (Eqn.2)$$

Recall that the (a, b, δ) triplet specifies a specific psychometric function, or probability of a correct response at any groove width, x: P_{cx}. In equation (2), we find the probability of the participant's data given a particular *a*-parameter by marginalizing over *b* and *δ*. Because we use a uniform prior on (a, b, δ), P(b, δ | a) is constant and subsumed in the proportion sign. P(d_i | a, b, δ) is determined by a binomial calculation on the participant's n_r right and n_w wrong answers at each groove width, x.

The second term on the right side of equation (1) is

$$P(a \mid M_0, C_1, \sigma) = (0.9)\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(a-\hat{a})^2}{2\sigma^2}\right) + (0.1)\frac{1}{(2mm)\sqrt{2\pi}}\exp\left(-\frac{(a-\hat{a})^2}{2(2mm)^2}\right) \quad (Eqn.3)$$

Equation (3) uses a robust (outlier-resistant) mixture model to calculate the probability of any *a*-parameter given M_0 with specific C_1 and σ. This model assumes that about 10% of participants may be outliers, and applies a large standard deviation (2 mm) to the outlier component of the mixture. The effect of this procedure is to protect inferences about model parameter values from being unduly influenced by occasional outlier participants.

Having evaluated Equation (1) for each participant's data, we next determine the probability of the entire data set, D, consisting of all participants' data ($D = \{d_i\}$) according to the model with a particular C_1 and σ:

$$P(D \mid M_0, C_1, \sigma) = \prod_i P(d_i \mid M_0, C_1, \sigma) \quad (Eqn.4)$$

Equation (4) assumes conditional independence among the participants' data, given C_1 and σ.

Finally, we calculate the marginal likelihood of the Model:

$$P(D \mid M_0) = \sum_{C,\sigma} P(D \mid M_0, C_1, \sigma) P(C_1, \sigma \mid M_0) = \frac{1}{N_c N_\sigma} \sum_{C,\sigma} P(D \mid M_0, C_1, \sigma)$$

Note that model M_0 assumes a uniform prior probability on (C_1, σ), which is simply the inverse of the number of (C_1, σ) pairs considered by the model.

Sex model (M_1)

According to the sex model, male and female thresholds belong to different Gaussian populations, the male mean being higher than the female mean, each with (shared) standard deviation σ. The expected 76% threshold (*a*-parameter) for participant i, then, is

$$\hat{a}_i = C_1 + C_2(g_i)$$

where C_2 is the (unknown) effect of sex on threshold, and *g* is a dummy variable:

$$g = \begin{cases} -0.5, female \\ +0.5, male \end{cases}$$

The model considers only positive values for C_2; that is, the model assumes that men have higher thresholds on average than women.

Following the same calculation sequence as for the null model, we find that:

$$P(d_i \mid M_1, C_1, C_2, \sigma) = \sum_a P(d_i \mid a) \bullet P(a \mid M_1, C_1, C_2, \sigma)$$

where

$$P(d_i \mid a) \propto \sum_{b,\delta} \prod_x \left[p_{cx}(a,b,\delta)\right]^{n_{rx}} \bullet \left[1 - p_{cx}(a,b,\delta)\right]^{n_{wx}}$$

and

$$P(a \mid M_1, C_1, C_2, \sigma) = (0.9)\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(a-\hat{a}_i)^2}{2\sigma^2}\right) + (0.1)\frac{1}{(2mm)\sqrt{2\pi}}\exp\left(-\frac{(a-\hat{a}_i)^2}{2(2mm)^2}\right)$$

Then,

$$P(D \mid M_1, C_1, C_2, \sigma) = \prod_i P(d_i \mid M_1, C_1, C_2, \sigma)$$

and

$$P(D \mid M_1) = \frac{1}{N_{c1}N_{c2}N_\sigma} \sum_{C_1, C_2, \sigma} P(D \mid M_1, C_1, C_2, \sigma)$$

Finger size model (M_2)

According to the finger size model, tactile threshold relates linearly to fingertip surface area. The expected 76% threshold (*a*-parameter) for participant i, then, is

$$\hat{a}_i = C_1 + C_2(s_i)$$

where C_2 is the (unknown) effect of finger size on threshold, and the variable *s*, derived by linear transformation of finger tip surface area, is equal to -0.5 for the smallest finger of the sample and + 0.5 for the largest. The model considers a range of possible values for C_2, all positive; that is, the model assumes that larger fingers have higher thresholds on average than smaller ones.

Following the same calculation sequence as for the previous models, we have:

$$P(d_i \mid M_2, C_1, C_2, \sigma) = \sum_a P(d_i \mid a) \bullet P(a \mid M_2, C_1, C_2, \sigma)$$

where

$$P(d_i \mid a) \propto \sum_{b,\delta} \prod_x \left[p_{cx}(a,b,\delta)\right]^{n_{rx}} \bullet \left[1 - p_{cx}(a,b,\delta)\right]^{n_{wx}}$$

and

$$P(a \mid M_2,C_1,C_2,\sigma) = (0.9)\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(a-\hat{a}_i)^2}{2\sigma^2}\right) + (0.1)\frac{1}{(2mm)\sqrt{2\pi}}\exp\left(-\frac{(a-\hat{a}_i)^2}{2(2mm)^2}\right)$$

Then,

$$P(D \mid M_2,C_1,C_2,\sigma) = \prod_i P(d_i \mid M_2,C_1,C_2,\sigma)$$

and

$$P(D \mid M_2) = \frac{1}{N_{c1}N_{c2}N_\sigma}\sum_{C_1,C_2,\sigma} P(D \mid M_2,C_1,C_2,\sigma)$$

Sex and finger size model (M_3)

According to the sex-and-finger-size model, tactile threshold relates linearly to fingertip surface area, but also differs between the sexes. The expected 76% threshold (*a*-parameter) for participant i, then, is

$$\hat{a}_i = C_1 + C_2(g_i) + C_3(s_i)$$

where C_2 is the effect of sex on threshold, and C_3 is the effect of finger size on threshold. The variables *g* and *s* are defined as previously, and the model considers only positive values for C_2 and C_3; that is, the model assumes one-sided effects, in which men have higher thresholds on average than women, and larger fingers have higher thresholds on average than smaller ones.

Following the same calculation sequence as for the previous models, we have:

$$P(d_i \mid M_3,C_1,C_2,C_3,\sigma) = \sum_a P(d_i \mid a) \bullet P(a \mid M_3,C_1,C_2,C_3,\sigma)$$

where

$$P(d_i \mid a) \propto \sum_{b,\delta}\prod_x \left[p_{cx}(a,b,\delta)\right]^{n_{rx}} \bullet \left[1 - p_{cx}(a,b,\delta)\right]^{n_{wx}}$$

and

$$P(a \mid M_3,C_1,C_2,C_3,\sigma) = (0.9)\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(a-\hat{a}_i)^2}{2\sigma^2}\right) + (0.1)\frac{1}{(2mm)\sqrt{2\pi}}\exp\left(-\frac{(a-\hat{a}_i)^2}{2(2mm)^2}\right)$$

Then,

$$P(D|M_3, C_1, C_2, C_3, \sigma) = \prod_i P(d_i \mid M_3, C_1, C_2, C_3, \sigma)$$

and

$$P(D|M_3) = \frac{1}{N_{c1}N_{c2}N_{c3}N_\sigma} \sum_{C_1, C_2, C_3, \sigma} P(D \mid M_3, C_1, C_2, C_3, \sigma)$$

Parameter Ranges and Sensitivity Analysis

The parameters within each model were assigned uniform prior probability densities over specified ranges (i.e., step-function priors). Calculations were done numerically, assigning discrete values for each parameter, evenly distributed within each range. All four models considered 15 values for C_1, ranging from 1.0 to 2.0 mm, and 15 values for σ, ranging from 0.05 to 0.55 mm.

Model 1 considered 21 values for C_2, ranging from 0.05 to 0.5 mm. The range 0.05-0.5mm was chosen based on previous research showing a sex difference in acuity of approximately 0.2 mm (Goldreich & Kanics, 2003, 2006).

Model 2 considered 21 values for C_2, ranging from 0.05 to 1.4 mm. Because no previous studies investigated the effect of finger size on tactile acuity, this range for C_2 was chosen to bracket the previously observed variability in tactile thresholds of participants of the same age and sex (Goldreich & Kanics, 2003, 2006).

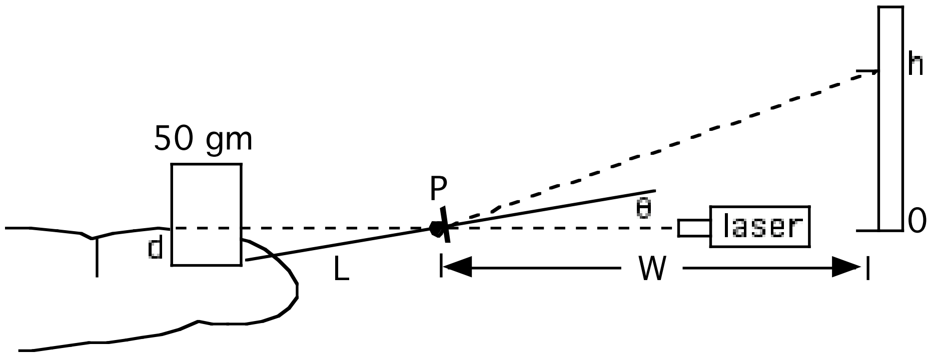
Model 3 used for C_2 the same values as did Model 1, and for C_3 the same C_2 values as did Model 2.

In order to assess the robustness of our results to reasonable variations in these prior assumptions, we repeated our analyses using different within-model parameter ranges. The Supplemental Table shows the Bayes factors that resulted. Note that in every case the finger size model is favored by the data.

Range for the Effect of Sex (mm)	Range for the Effect of Area (mm): 0.68	1.35	2.7
0.23	Bayes Factors: Sex: 4.13 **Finger Size: 213.29** Sex & Finger Size: 80.75	Bayes Factors: Sex: 4.13 **Finger Size: 170.86** Sex & Finger Size: 48.94	Bayes Factors: Sex: 4.13 **Finger Size: 85.46** Sex & Finger Size: 24.53
0.45	Bayes Factors: Sex: 2.25 **Finger Size: 213.29** Sex & Finger Size: 43.16	Bayes Factors: Sex: 2.25 **Finger Size: 170.86** Sex & Finger Size: 26.33	Bayes Factors: Sex: 2.25 **Finger Size: 85.46** Sex & Finger Size: 13.2
0.9	Bayes Factors: Sex: 1.15 **Finger Size: 213.29** Sex & Finger Size: 24.17	Bayes Factors: Sex: 1.15 **Finger Size: 170.86** Sex & Finger Size: 14.97	Bayes Factors: Sex: 1.15 **Finger Size: 85.46** Sex & Finger Size: 7.5

Supplemental Table. Sensitivity analysis showing the effect of varying the parameter ranges over which the models distributed prior probability. Bayes factors for Models 1, 2, and 3 relative to the null are reported in each cell. Model 2, the finger size model, is favored by the greatest Bayes factor (bold) in each case.

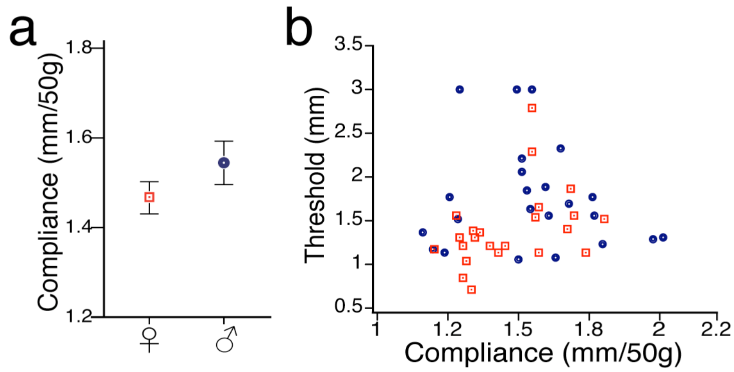
Supplemental Figures



Supplemental Figure 1. Skin compliance measurement apparatus and procedure. The dominant hand rested in supine position on the platform of a lab jack. The hand was restrained gently with straps, and the index fingernail attached to the platform with double-sided tape. The platform was raised until the index finger skin contacted a 0.5-inch-diameter smooth circular surface attached to the end of a rotating rod at a distance *L* = 11 cm from the pivot point (P). A first-surface mirror (Edmund Optics, Barrington, NJ) attached to the rod at P reflected a laser beam onto a wall at distance *W* = 413 cm. The height of the lab jack platform was adjusted so that the laser beam projected to position zero on the wall. A mass, *M* = 50 gm, was then placed in a receptacle overlying the finger-contact surface. Indentation of the skin caused rotation of the rod by angle θ. Since the angle of incidence between the light beam and mirror equaled the angle of reflection, rotation of the mirror by θ caused the laser beam to project at angle twice θ from the horizontal. We derived the displacement (*d*) of the skin, from the height of the laser dot (*h*) on the wall, given the constants *L* and *W*:

$$d = L \sin\left[0.5 ArcTan\left(\frac{h}{W}\right)\right]$$

We made five compliance measurements on each participant, waiting each time for six seconds after placement of the mass for the skin to reach steady-state indentation. The sensitivity of the system was such that the slight increase in fingertip blood volume caused by the heartbeat (the pulse in the fingertip produced by the systolic phase of the cardiac cycle) often produced a noticeable decrease in the height, *h*, of the laser beam dot. In such cases, we took the measurements in the diastolic phase (maximum laser dot height of the pulse cycle, corresponding to lowest fingertip blood volume).



Supplemental Figure 2. Skin compliance data. **(a)** Skin compliance by sex. Compliance was not significantly greater in women than in men (unpaired t-test, p = 0.9). **(b)** Scatterplot of GOT threshold v. skin compliance. Contrary to the prediction of Hypothesis 1, GOT threshold did not decrease significantly with increasing skin compliance (correlation, p = 0.8). In fact, the non-significant trend was for threshold to increase with compliance. Multiple linear regression of GOT threshold (dependent variable) against skin compliance and fingertip surface area (independent variables) showed a significant effect of fingertip area (p = 0.02) but again no effect of skin compliance (p = 0.7). Women: ; men: .